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Part 1. Stationary states

K. Verhulst, R. Cardinaels, P. Moldenaers

Lab for Applied Rheology and Polymer Processing

Department of Chemical Engineering

KU Leuven

Willem de Croylaan 46, Box 2423, B-3001 Leuven, Belgium

Paula.Moldenaers@cit.kuleuven.be

Y. Renardy, S. Afkhami

Department of Mathematics and ICAM

Virginia Polytechnic Institute and State University

460 McBryde Hall, Blackburg , VA24061-0123, USA

Renardyy@aol.com

Final accepted draft

Cite as: K. Verhulst, R. Cardinaels, P. Moldenaers, Y. Renardy, S. Afkhami, J. Non-Newt. Fluid Mech., 156(1-2), pp. 29-43 (2009)

The original publication is available at:

<http://www.sciencedirect.com/science/article/pii/S037702570800116X>

Influence of viscoelasticity on drop deformation and orientation in shear flow

Part 1: Stationary states

Kristof Verhulst^a Ruth Cardinaels^a Paula Moldenaers^a

^a*Katholieke Universiteit Leuven, Department of Chemical Engineering, W. de Croylaan 46 - B 3001 Heverlee (Leuven), Belgium*

Yuriko Renardy^b Shahriar Afkhami^b

^b*Department of Mathematics and ICAM, 460 McBryde Hall, Virginia Polytechnic Institute and State University, Blacksburg VA 24061-0123, USA*

Abstract

The influence of matrix and droplet viscoelasticity on the steady deformation and orientation of a single droplet subjected to simple shear is investigated microscopically. Experimental data are obtained in the velocity-vorticity and velocity-velocity gradient plane. A constant viscosity Boger fluid is used, as well as a shear-thinning viscoelastic fluid. These materials are described by means of an Oldroyd-B, Giesekus, Ellis, or multi-mode Giesekus constitutive equation. The drop-to-matrix viscosity ratio is 1.5. The numerical simulations in 3D are performed with a volume-of-fluid algorithm and focus on capillary numbers 0.15 and 0.35. In the case of a viscoelastic matrix, viscoelastic stress fields, computed at varying Deborah numbers, show maxima slightly above the drop tip at the back and below the tip at the front. At both capillary numbers, the simulations with the Oldroyd-B constitutive equation predict the experimentally observed phenomena that matrix viscoelasticity significantly suppresses droplet deformation and promotes droplet orientation, two effects that saturate at high Deborah numbers. Experimentally, this corresponds to decreasing the droplet radius with other parameters unchanged. At the higher capillary and Deborah numbers, the use of the Giesekus model with a small amount of shear-thinning dampens the stationary state deformation slightly and increases the angle of orientation. Droplet viscoelasticity on the other hand hardly affects the steady droplet deformation and orientation, both experimentally and numerically, even at moderate to high capillary and Deborah numbers.

Key words: drop deformation, Oldroyd-B model, volume-of-fluid method, blend morphology, viscoelasticity

1 Introduction

Dilute polymeric blends commonly form a droplet-matrix interface [1, 2]. The final material properties are significantly influenced by the deformation, break-up and coalescence of droplets during flow. Control of these processes is therefore essential in the development of high performance blends. The investigation of single droplet dynamics in simple shear is a contribution toward this goal. In fact, theoretical and experimental studies on the single droplet problem recently include the effects of component viscoelasticity [3]. Perturbation theories for small deformation [4, 5] predict that viscoelasticity hardly affects the steady droplet deformation at low flow intensity. Steady droplet orientation towards the (shear) flow direction is predicted to be highly promoted by matrix viscoelasticity. The effect of droplet viscoelasticity on the orientation of the droplet is less pronounced. Several experimental studies confirm these trends [6, 7, 8].

The small deformation theories are modified and extended to handle larger droplet deformations in recent phenomenological models. In the case of steady droplet deformation, quantitative agreement is found for low to moderate droplet deformations. At high droplet deformations, the discrepancy with experimental results is more pronounced. [8, 9, 10, 11, 12, 13, 14].

In planar extensional flow [15] and uniaxial elongational flow [16], matrix elasticity promotes droplet deformation at stationary states for viscosity ratios ($\lambda = \text{drop to matrix ratio}$) less than or equal to one. In the case of droplet elasticity, the opposite is found [16, 17]. For $\lambda > 1$, experiments on planar extensional flow with various non-Newtonian droplet systems result in stationary shapes that resemble corresponding Newtonian-Newtonian systems [18]. Less attention has been given to the study of simple shear. Experimental results of Guido *et al.* [7] demonstrate that matrix viscoelasticity suppresses droplet deformation at high flow intensities at viscosity ratios 0.1, 1 and 4.7. This result is confirmed by Verhulst *et al.* [8] at viscosity ratio 0.75 and is in qualitative agreement with the predictions of the phenomenological models [9, 10, 11]. On the contrary, several authors conclude the opposite, *i.e.* matrix elasticity enhances droplet deformation [19, 20, 21]. The numerical investigation of Yue *et al.* [22] clarifies the interaction between the various stress components and pressure acting on the surface of the droplet, and explains the differences according to the level of matrix elasticity. In addition, Verhulst *et al.* [8] demonstrate that similar materials, studied at moderate to high shear rates, can yield different steady droplet deformations under the same experimental conditions; *i.e.* when the same dimensionless parameters (borrowed

¹ Corresponding author:

Email address: renardyy@aol.com Y. Renardy

from small deformation theory) are studied for different materials.

Experimental studies on the deformation of a viscoelastic droplet in a Newtonian matrix have only been performed at a viscosity ratio of one in the papers of Lerdwijitjarud *et al.* [23, 24] and Sibillo *et al.* [13]. In the latter, the experimental results are compared with a model equation. Numerical simulations are conducted in [25] for an Oldroyd-B droplet in a Newtonian matrix under simple shear. The drop is found to deform less as the Deborah number increases, while at high capillary numbers, the deformation increases with increasing Deborah number. A first-order ordinary differential equation is used as a phenomenological model [25]. It describes an overdamped system, in which the viscous stretching force is proportional to the shear rate, a damping term is proportional to viscosity, and a restoring force is proportional to the first normal stress difference. The latter creates an elastic force which acts to eventually decrease deformation, but it predicts greater decrease than observed in the numerical results. Moreover, the model can not predict the critical curve in the Ca vs. λ parameters for the Stokes regime, nor the transient overshoot which results from strong initial conditions [26], or predict results for a Newtonian drop in a viscoelastic matrix. The suppression of deformation with increasing drop elasticity is also noted in the theoretical and numerical studies of [9, 22, 27, 28].

In this paper, the influence of both matrix and droplet viscoelasticity on the steady deformation and orientation of a single droplet subjected to a homogeneous shear flow is investigated at a viscosity ratio of 1.5. The study is performed for a broad range of the relevant dimensionless parameters, which allows the examination of the dependence on matrix elasticity in 3D at high Deborah numbers. The 2D study of [22] finds a numerically small non-monotonic dependence of stationary state deformation on the matrix Deborah number, which is not noticeable in 3D for the specific parameters of this study. Throughout this paper, the experimental results are compared with three-dimensional simulations performed with a volume-of-fluid algorithm for viscoelastic liquid-liquid systems.

2 Numerical simulations

The governing equations are as follows. The liquids are density-matched. For each liquid, the solvent viscosity is denoted η_s , polymeric viscosity η_p , total viscosity $\eta = \eta_s + \eta_p$, relaxation time τ , shear rate $\dot{\gamma}$, and the initial elastic modulus $G(0) = \eta_p/\tau$. Additional subscripts ‘d’ and ‘m’ denote the drop and matrix liquids. The governing equations include incompressibility and momentum transport:

$$\begin{aligned}\nabla \cdot \mathbf{u} &= 0, \\ \rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) &= \nabla \cdot \mathbf{T} - \nabla p + \nabla \cdot (\eta_s (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)) + \mathbf{F},\end{aligned}\tag{1}$$

where \mathbf{T} denotes the extra stress tensor. The total stress tensor is $\underline{\tau} = -p\mathbf{I} + \mathbf{T} + \eta_s[\nabla \mathbf{u} + (\nabla \mathbf{u})^T]$. Each liquid is identified with a color function,

$$C(\mathbf{x}, t) = \begin{cases} 0 & \text{in the matrix liquid} \\ 1 & \text{in the drop} \end{cases}\tag{2}$$

which advects with the flow. The position of the interface is given by the discontinuities in the color function. The interfacial tension force is formulated as a body force

$$\mathbf{F} = \Gamma \tilde{\kappa} \mathbf{n} \delta_S, \quad \tilde{\kappa} = -\nabla \cdot \mathbf{n},\tag{3}$$

where Γ denotes the surface tension, \mathbf{n} the normal to the interface, δ_S the delta-function at the interface, and $\tilde{\kappa}$ the curvature. In (3), $\mathbf{n} = \nabla C / |\nabla C|$, $\delta_S = |\nabla C|$.

The drop and matrix liquids are governed by the Giesekus constitutive equations, which has had reasonable success in comparisons of two-layer channel flow with actual data [29]. One feature that distinguishes the Giesekus model from others is the non-zero second normal stress difference N_2 . This is relevant to the flow of two immiscible liquids because it has been shown that a discontinuity in N_2 affects interfacial stability [30]. In one limit, the Giesekus model reduces to the simpler Oldroyd-B constitutive equation. The experimental data which are addressed in this paper are as a first approximation, Oldroyd-B liquids [8]. However, the Oldroyd-B model is a difficult one to implement because it overpredicts the growth of stresses at large deformation rates and lead to numerical instability. The Giesekus constitutive equation is

$$\begin{aligned}\tau \left(\frac{\partial \mathbf{T}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{T} - (\nabla \mathbf{u}) \mathbf{T} - \mathbf{T} (\nabla \mathbf{u})^T \right) &+ \mathbf{T} + \tau \kappa \mathbf{T}^2 \\ &= \tau G(0) (\nabla \mathbf{u} + (\nabla \mathbf{u})^T).\end{aligned}\tag{4}$$

The dimensionless parameters are the viscosity ratio (based on total viscosities) $\lambda = \frac{\eta_d}{\eta_m}$, a capillary number $Ca = \frac{R_0 \dot{\gamma} \eta_m}{\Gamma}$, a Weissenberg number per fluid $We = \dot{\gamma} \tau$, and retardation parameter per fluid $\beta = \frac{\eta_s}{\eta}$. A Reynolds number based on the matrix liquid $Re = \frac{\rho \dot{\gamma} R_0^2}{\eta_m}$ is in the range .01 to .05, chosen small so that inertia is negligible.

Alternatively, let Ψ_1 denote the first normal stress coefficient, equivalent to $2\eta_p \tau$, and define the Deborah number by

$$\widetilde{De} = \frac{\Psi_1 \Gamma}{2R_0 \eta^2}; \quad (5)$$

i.e., $\widetilde{De}_d = (1 - \beta_d)We/(\lambda Ca)$, and $\widetilde{De}_m = (1 - \beta_m)We/Ca$. Numerical and experimental results in later sections are presented in terms of the Deborah numbers, and dimensionless capillary time

$$\frac{t\Gamma}{\eta_m R_0} = \frac{t\dot{\gamma}}{Ca}, \quad (6)$$

hereinafter denoted \hat{t} . The rescaled Giesekus parameter

$$\tau \kappa G(0) \quad (7)$$

is relabeled $\hat{\kappa}$. The physically viable range is $0 \leq \hat{\kappa} < 0.5$ [30]. The Oldroyd-B model is $\hat{\kappa} = 0$.

The governing equations are discretized with the volume-of-fluid (VOF) method given in Ref. [26]. The interfacial tension force (3) is approximated by either the continuum surface force formulation (CSF) or the parabolic representation of the interface for the surface tension force (PROST). The reader is referred to Refs. [31, 32, 28] for these algorithms. Both VOF-PROST and VOF-CSF codes are parallelized with OpenMP. The efficiency of the parallelization for the Newtonian part of the code is discussed in [33]; the viscoelastic part has analogous properties. A typical computation presented in this paper is $\Delta x = R_0/16$, $\Delta \hat{t} = 0.00005/Ca$, $L_x = 16R_0$, $L_y = 8R_0$, $L_z = 8R_0$, for which one timestep takes 1.2 seconds with 64 processors on the SGI Altix 3700 supercluster at Virginia Tech. A simulation from dimensionless capillary time 0 to 11, with 64 nodes takes roughly 25 hours.

2.1 Boundary conditions

The computational domain is denoted $0 \leq x \leq L_x, 0 \leq y \leq L_y, 0 \leq z \leq L_z$. The boundaries at $z = 0, L_z$ are walls which move with speeds $\pm U_0$. This results in the velocity field $(U(z), 0, 0)$ in the absence of the drop, where $U(z) = U_0(2z - L_z)/L_z$. The shear rate is $\dot{\gamma} = U'(z) = 2U_0/L_z$. Spatial periodicity is imposed in the x and y directions, at $x = 0, L_x$ and $y = 0, L_y$, respectively. Additional boundary conditions are not needed for the extra stress components. For computational efficiency, L_x, L_y and L_z are chosen to minimize the effect of neighboring drops and that of the walls. Typically, the distance between the walls is eight times the drop radius, as is the spanwise period, and the period in the flow direction is chosen dependent on drop extension. In these and prior (Newtonian) simulations, the influence of the

boundaries is negligible under these circumstances. Experimental results on the effect of confinement are consistent with this [34].

2.2 Initial conditions

The drop is initially spherical with radius R_0 . The top and bottom walls are impulsively set into motion from rest. This requires that the viscoelastic stress tensor and velocity are initially zero. Since the velocity field is essentially governed by a parabolic PDE close to Stokes flow, the velocity adjusts immediately to simple shear so that whether the initial velocity field is zero or simple shear makes no difference in the numerical simulations. The initial viscoelastic stress values do, on the other hand, influence drop deformation. For example, if the drop were placed in a pre-existing shear flow, the initial viscoelastic stress is equal to the values which would prevail in the corresponding simple shear flow with the given shear rate. Figure 4 of [26] shows that a drop placed in an already established flow immediately experiences a large viscoelastic stress and the deformation may overshoot. On the other hand, if the matrix fluid is viscoelastic with zero initial viscoelastic stress, then even with an established velocity field, it starts out with lower viscous stress due to the absence of polymer viscosity. This lowers the magnitude of stress in the matrix fluid, which pulls the drop more gently.

2.3 Drop diagnostics

We report the drop diagnostics with the same notation as in [8, 35]. The slice through the center in the $x - z$ plane, or the velocity-velocity gradient plane, provides the drop length L and breadth B . The angle of inclination to the flow direction is denoted θ . When viewed from above the drop, the slice through the center of the drop in the $x - y$ plane, or the velocity-vorticity plane, gives the drop width W and length L_p . The Taylor deformation parameters are $D = (L - B)/(L + B)$ and $D_p = (L_p - W)/(L_p + W)$. The viscoelastic stress fields displayed in this paper are generated from contour values of the trace of the extra stress tensor, $\text{tr}(\mathbf{T})$. This is directly proportional to the extension of the polymer molecules, and is meaningful to plot from the direct numerical simulations. This quantity is directly related to experimental measurements on stress fields with birefringence techniques. Contour values are given on the numerical plots in order to provide a comparison among the plots.

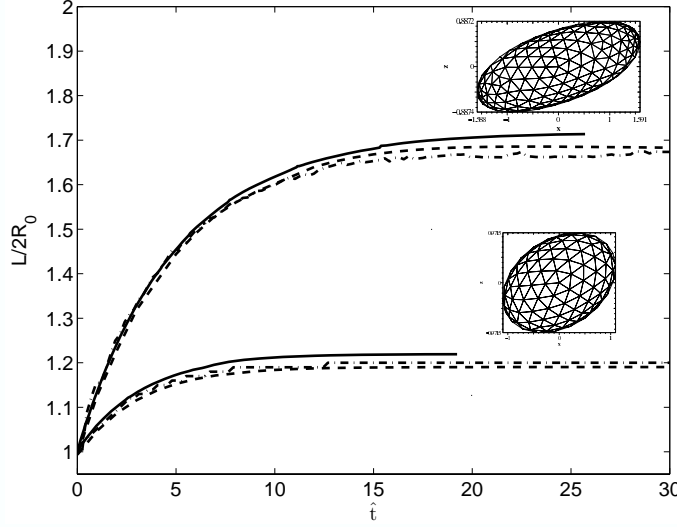


Fig. 1. Newtonian reference system with viscosity ratio 1.5 (system 4 of table 2). Experimental data ($-\cdot-$) at $Ca = 0.156$ (lower), 0.363 (upper) are compared with numerical simulations with the boundary integral code of [36] ($--$) and VOF-CSF ($—$).

2.4 Accuracy

Computational accuracy for the Oldroyd-B portion is discussed in [28] for the 2D code, and in [26] for the 3D version. Figure 1 is a comparison of the experimental data ($-\cdot-$) on the sideview length for the Newtonian reference system 4 of table 2. Two numerical methods, the VOF-CSF ($-$) and the boundary integral scheme of Ref. [36], are used to provide quantitative agreement.

Tests for convergence with spatial and temporal refinements are performed to decide on optimal meshing. Table 1 shows deformation and angle of inclination for an Oldroyd-B drop in Newtonian matrix with the CSF formulation at $Ca = 0.35$, $\widetilde{De}_d = 2.6$, $\lambda = 1.5$ and $Re = 0.05$. The computational box is $L_x = 16R_0$, $L_y = 8R_0$, $L_z = 8R_0$. The results are also close for the PROST algorithm; since PROST is more CPU intensive, CSF is used for later results.

Figure 2 shows viscoelastic stresses computed for the vertical cross-section through the center of a Oldroyd-B drop. The first row is $\hat{t} = 5$, the second row is $\hat{t} = 10$ and the third is $\hat{t} = 15$. With $\Delta t \dot{\gamma} = 0.0001$, the mesh is varied from $\Delta x = R_0/8$ for the first column, $\Delta x = R_0/12$ for the middle column and $R_0/16$ for the third column. The middle column is chosen as optimal:

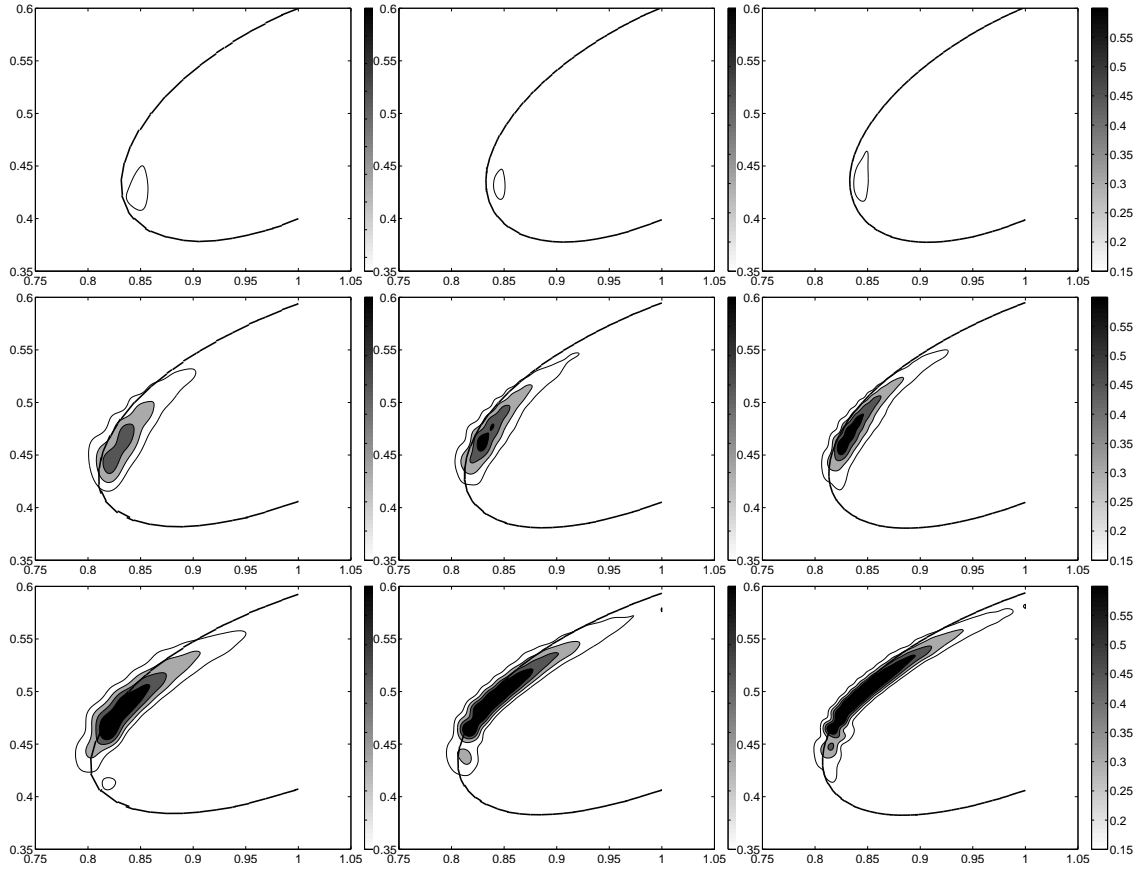


Fig. 2. Contour plots for viscoelastic stress for a Oldroyd-B drop in Newtonian matrix, $Ca = 0.35$, $\widetilde{De}_d = 2.6$, $\lambda = 1.5$, $\hat{t} = 5$ (row 1), 10 (row 2), 15 (row 3); refinement left to right $\Delta x = R_0/8, R_0/12, R_0/16$.

$\Delta x = R_0/12$ for $Ca = 0.35$, $\widetilde{De}_d = 2.6$, $\lambda = 1.5$. The temporal mesh is also varied to optimize the timestep. The stresses vary steeply close to the interface and the contours are constructed from an interpolation of the values from the simulations. This procedure may contribute to some inaccuracy in the shape of the contours, and also that they cross the interface. It is clear therefore that mesh refinement reduces these features. A sensitivity study on the Reynolds number is performed at $Ca = 0.35$, $\widetilde{De}_d = 0$, $\widetilde{De}_m = 1.8$, $\beta_m = 0.68$, $\lambda = 1.5$, $L_x = 16R_0$, $L_y = 8R_0$, $L_z = 8R_0$, $\Delta x = R_0/12$. The following produce similar results: (i) $Re = 0.05$, $\Delta \hat{t} = 0.00003$, (ii) $Re = 0.01$, $\Delta \hat{t} = 0.00003$, (iii) $Re = 0.05$, $\Delta \hat{t} = 0.000014$, all with CSF, and (iv) $Re = 0.05$, $\Delta \hat{t} = 0.00003$ with PROST. $Re = 0.05$ is chosen as optimal.

3 Materials and experimental methods

Table 2 lists the interfacial tension and viscosity ratio for the droplet-matrix systems used in this study. The first two blends carry a viscoelastic droplet

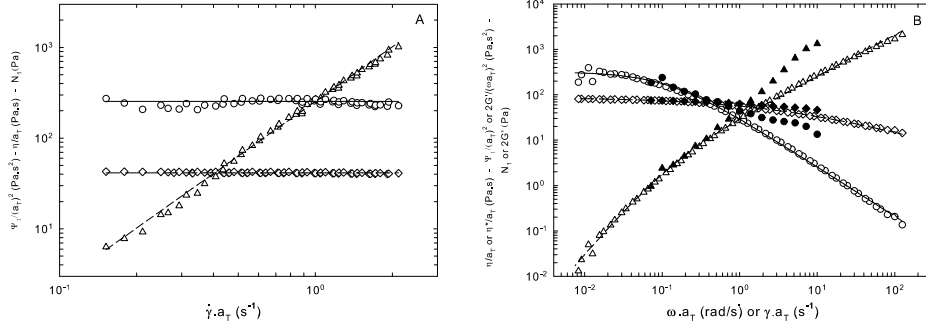


Fig. 3. Rheological characterization of the viscoelastic fluids at a reference temperature of 25 °C. (a) PIB Boger fluid BF2. First normal stress difference: Δ , viscosity: \diamond , first normal stress coefficient: \circ ; Lines are the Oldroyd-B model. (b) Branched PDMS BR16. Open symbols are dynamic data, $2G' / (\omega a_T)^2$: \circ , dynamic viscosity: \diamond , $2G''$: Δ ; filled symbols are steady shear data, first normal stress coefficient: \bullet , viscosity: \blacklozenge , first normal stress difference: \blacktriangle ; Lines are the Ellis model.

phase; the third blend contains a viscoelastic matrix phase; and the fourth one acts as the reference system, containing only Newtonian components.

The viscoelastic material is either a branched polydimethylsiloxane (PDMS) named BR16, or a polyisobuthylene Boger fluid (PIB) named BF2 which is described in detail in [8]. Preparation of the Boger fluid requires the addition of 0.2 weight percentage of a high molecular weight rubber (Oppanol B200) to a Newtonian PIB (Infineum S1054), which acts as the non-volatile solvent. As Newtonian materials, various mixtures of linear PDMS (Rhodorsil) or PIB (Infineum or Parapol) are used in order to obtain the desired viscosity ratio of 1.5. In addition, the PDMS used as the matrix fluid is saturated with a low molecular weight polyisobuthylene (Indopol H50). This saturation step is necessary to avoid diffusion of PIB molecules, which would lead to droplet shrinkage and a time-dependent interfacial tension [37]. The interfacial tension in table 2 is measured with two independent methods, which both agree to within experimental error: (i) fitting the droplet deformation at small flow intensity to the second-order theory of Greco [4] and (ii) the pendant drop method.

The rheology of the blend components is discussed in detail in section III.A of Ref. [8]. Briefly, all Rhodorsil PDMS mixtures, the Parapol 1300, and the Infineum mixture are Newtonian in the results of this paper. The steady shear rheological data of the PIB Boger fluid at 25 °C is shown in fig. 3a. The viscosity and first normal stress coefficient are clearly constants. Thus, the Oldroyd-B constitutive model is appropriate to describe the steady shear rheology, where the solvent viscosity equals that of the non-volatile solvent (Infineum S1054).

Figure 3b shows the rheological data of the branched PDMS, clearly displaying shear-thinning behavior. The Cox-Mertz rule is valid at the shear rates applied in the droplet deformation experiments. Hence, to describe the rheology of the branched PDMS, the dynamic data are selected and fitted with an Ellis model [38],

$$\frac{x}{x_0} = \frac{1}{1 + k\omega^{(1-n)}}, \quad (8)$$

where x_0 , k and n are fitting parameters and ω is the oscillation frequency. The resulting rheological parameters of all components at the temperatures used in the droplet deformation experiments are summarized in table 3.

Droplet deformation experiments with low to moderate Deborah numbers (up to 2) are performed with a counter rotating plate-plate device which is described in detail in section III.B of Ref. [8]. The experimental protocol and image analysis are detailed in section III.C[8]. Digital images are analyzed in the velocity-vorticity plane (top view) and velocity-velocity gradient plane (side view). The major and minor axes of the deformed droplet in the velocity-vorticity plane are obtained by fitting an equivalent ellipse to the drop contour as described in Ref. [39].

Additional droplet deformation experiments are performed with a Linkam CSS 450 shear cell [40]. The optical train consists of a bright light microscope (Leitz Laborlux 12 Pol S) and a Hamamatsu (Orka 285) digital camera, allowing higher magnifications and better resolution as compared with the counter rotating setup. Hence, smaller droplets can be studied, permitting droplet deformation experiments with Deborah numbers up to 20. Observations are however limited to the velocity-vorticity plane. Moreover, in the Linkam shear cell only the bottom plate rotates, so no stagnation plane exists and the studied droplet moves out of the observation area during flow. Therefore, dilute blends are studied in this apparatus. These blends are obtained by mixing 0.1 weight percentage of the dispersed phase into the matrix material using a spatula, resulting in countless droplets with a diameter less than 1 μm . The blend is deaerated in a vacuum oven and thereafter pre-sheared in the Linkam apparatus at a shear rate of 0.1 s^{-1} for 48 hours. The resulting morphology consists of uniformly distributed droplets with a radius between 10 - 20 μm , for which hydrodynamic and confinement effects (gap of 300 μm) can be excluded. During the droplet deformation experiments, a droplet is examined when entering the field of view, after which the flow is stopped. The droplet relaxes, thereby allowing accurate measurement of its initial radius at rest.

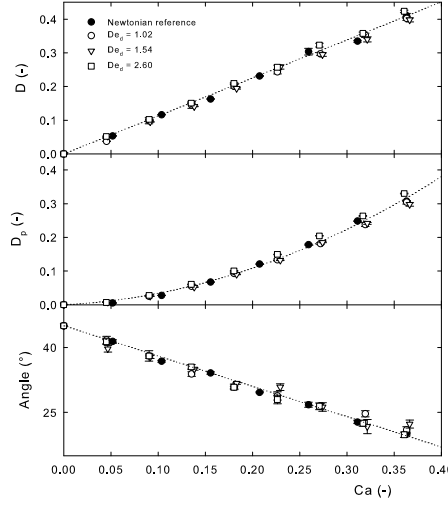


Fig. 4. Steady state droplet deformations and orientation. Open symbols: Boger fluid droplet - Newtonian matrix system (System 1 of table 2) at various \widetilde{De}_d ; filled symbols: Newtonian reference system.

4 Results and discussion

4.1 Boger fluid droplet system

The effects of droplet viscoelasticity on the droplet deformation and orientation are systematically studied over a wide range of Deborah and capillary numbers and thus contribute to the scarce data on viscoelastic droplet systems. Figure 4 shows the stationary droplet deformations D and D_p , and orientation of a Boger fluid droplet in a Newtonian matrix (System 1, table 2). The Newtonian/Newtonian reference (System 4, table 2) is also plotted for comparison. The range of Deborah number is achieved by varying the radius of the droplet. The counter-rotating setup was used, yielding the complete 3D picture of the deformed droplet. The figure shows hardly any difference between the elastic droplet system (open symbols) and the Newtonian/Newtonian reference system (filled symbols) at the same capillary number, beyond what one expects as the range of small deformation. Moreover, an increase in the Deborah number, *i.e* elasticity, does not result in any change in the steady deformation and orientation, at least within experimental error. Thus, even for Deborah numbers up to almost 3, which represents strong elasticity beyond the small deformation limit, the elastic drop behaves like a Newtonian one.

Numerical simulations and experimental data are compared in fig. 5 at $\widetilde{De}_d = 1.54$ for $Ca = 0.14$ and 0.32 . At the lower capillary number, the viscoelastic droplet simulation is similar to the Newtonian droplet simulation. For the higher Ca , the viscoelastic droplet simulation and data show less deformation

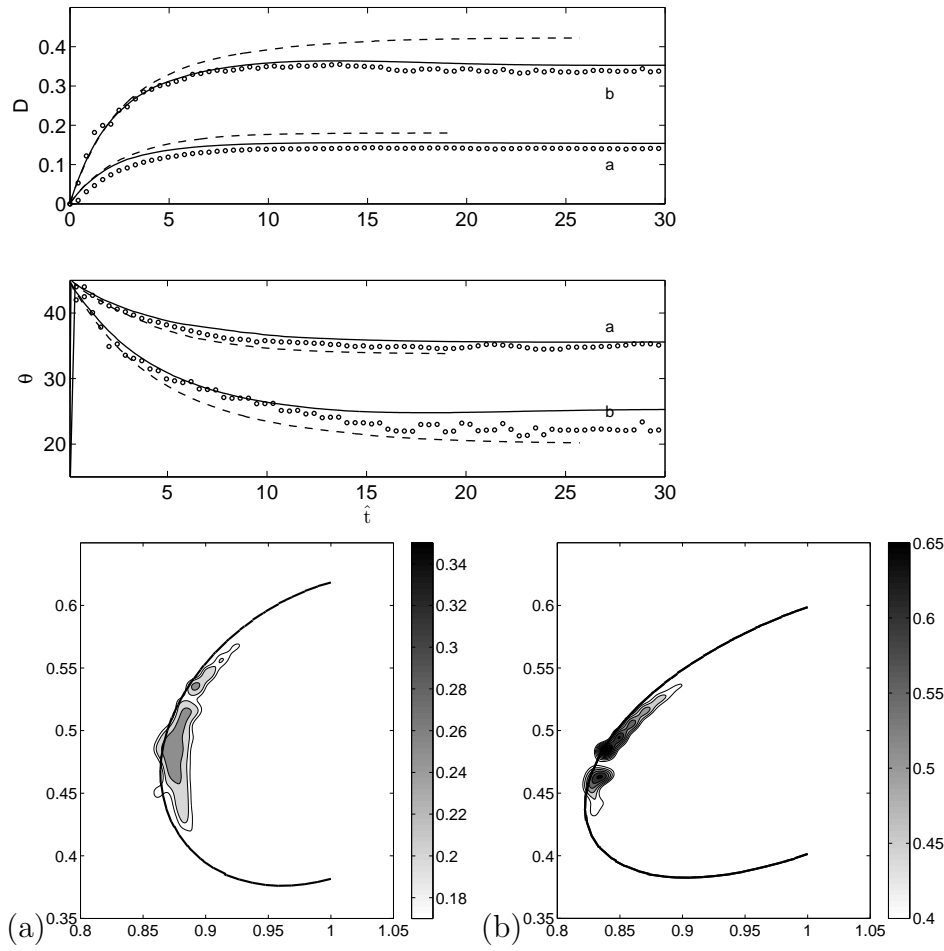


Fig. 5. Side view deformation, 3D VOF-CSF simulation (—) and experimental data (\circ) at fixed $\widetilde{De}_d = 1.54$, for varying $Ca =$ (a) 0.14; (b) 0.32, and Newtonian CSF simulation (---). The contours for viscoelastic stresses at stationary states are given.

than the Newtonian simulation. For fixed Deborah number, the magnitude of viscoelastic stress increases with capillary number. The location of the maximum viscoelastic stress migrates to slightly above the drop tip at the back, and slightly below the drop tip at the front when stationary state is reached.

Figure 6a shows the steady droplet deformation of the Boger fluid droplet system against the capillary number at higher Deborah numbers (\widetilde{De}_d up to almost 20), measured with the Linkam shear cell. This setup only allows observations in the velocity-vorticity plane, therefore only information on the axes W and L_p is presented. Figure 6a also includes the results at smaller \widetilde{De}_d , measured with the counter-rotating setup; and the Newtonian reference system, all at a viscosity ratio of 1.5. It is clear that, even at these very large Deborah numbers, the effect of droplet elasticity is insignificant. One could argue on the basis of figure 6a that at the large \widetilde{De}_d numbers the droplet elasticity has the tendency to suppress the droplet deformation by a small

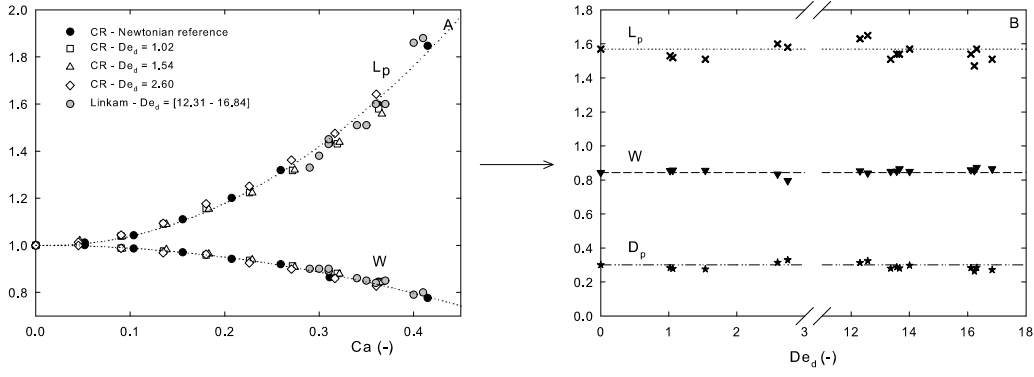


Fig. 6. Steady state droplet deformation of the Boger fluid droplet system at various \widetilde{De}_d numbers. (a) Top view results as a function of the capillary number; open symbols: counter rotating experiments; filled symbols: Newtonian reference system; gray symbols: Linkam experiments. (b) Top view results at fixed $Ca = 0.35$; lines denote Newtonian steady state deformation.

amount. But, by presenting the data as shown in figure 6b at a constant capillary number of 0.35, well beyond the range of small deformation, it is obvious that if any effect of droplet elasticity were present on L_p or W , it is within experimental error. This conclusion is even more unmistakable when plotting the deformation parameter D_p (see fig. 6b), where no effect of droplet elasticity is seen, even at the very high Deborah numbers. These results suggest that the droplet orientation at higher \widetilde{De}_d numbers is also hardly affected by droplet elasticity.

Numerical simulations for fig. 6 are conducted at a fixed $Ca = 0.35$ with varying Deborah numbers. Table 4 shows the steady state drop diagnostics from VOF-CSF simulations at $Ca = 0.35$, $\widetilde{De}_d = 1.02, 2.6, 4, 12.31$. These indicate a slight elongation with increasing \widetilde{De}_d .

Figure 7 shows steady state contours of elastic stress for $\widetilde{De}_d = 1.02, 2.6, 12.31$. The magnitude of the elastic stresses increases and their location shift to narrower areas along the interface as \widetilde{De}_d increases. The velocity fields are similar to the Newtonian reference system, and streamlines are shown for the $\widetilde{De}_d = 12.31$. The flow inside the drop is a recirculation, and the accompanying shear rate is different from the imposed shear rate outside the drop and moreover, it is not uniform but depends on the shape of the droplet. In a recirculating flow, the flow does not exhibit much stretching compared with the shear flow in the matrix. Therefore, viscoelastic stresses inside the drop are much smaller than those obtained outside the droplet in a viscoelastic matrix. Thus, the elongational component of the flow is weak, and correspondingly elastic effects; this is reminiscent of other recirculating flows, such as the corner eddies in contraction flow. In particular, the 4:1 contraction flow is known to have corner vortices with only weak elastic stresses [41]. The main differ-

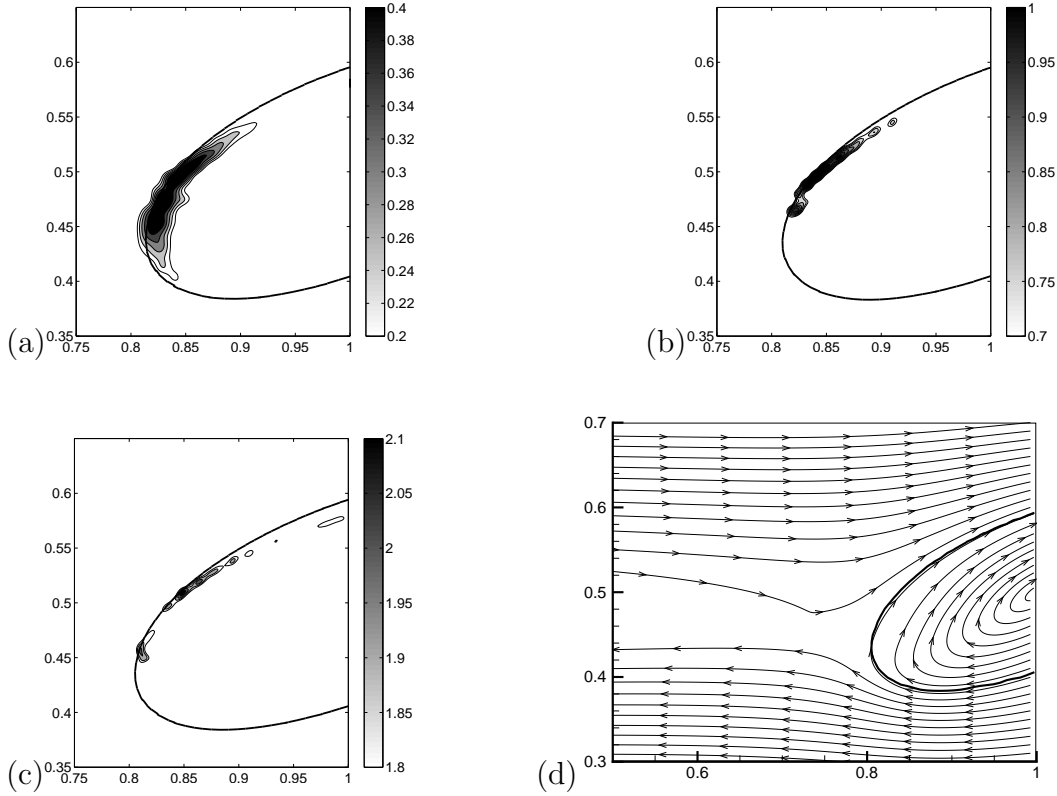


Fig. 7. Steady state viscoelastic stress contours through the vertical cross-section of the drop at $Ca = 0.35$, (a) $\widetilde{De}_d = 1.02$, (b) 2.6, (c) 12.31; (d) stream lines for $\widetilde{De}_d = 12.31$.

ence between these and the drop is that the outer contraction flow determines their sizes, while in the drop system, the size of the recirculation zone is given. At fixed capillary number, the position of maximum viscoelastic stress is seen to move slightly upwards at the back of the drop and also downwards at the front. Since the maxima are not at the drop tips, they promote rotation, and prevent the drop from elongating further.

4.2 Shear-thinning viscoelastic droplet system

In this section, the additional effects of shear-thinning behavior of the droplet fluid are systematically studied at both low and high shear flow intensities. Figure 8, for example, shows the steady deformation and orientation for the viscoelastic shear-thinning droplet (system 2 of table 2) as a function of the capillary and Deborah number. The Newtonian/Newtonian reference system is also plotted for comparison. The applied Deborah numbers are calculated using the zero-shear values of viscosity and first normal stress difference. The steady droplet shapes observed in the velocity-vorticity plane and the velocity-velocity gradient plane do not display any difference with the deformation ob-

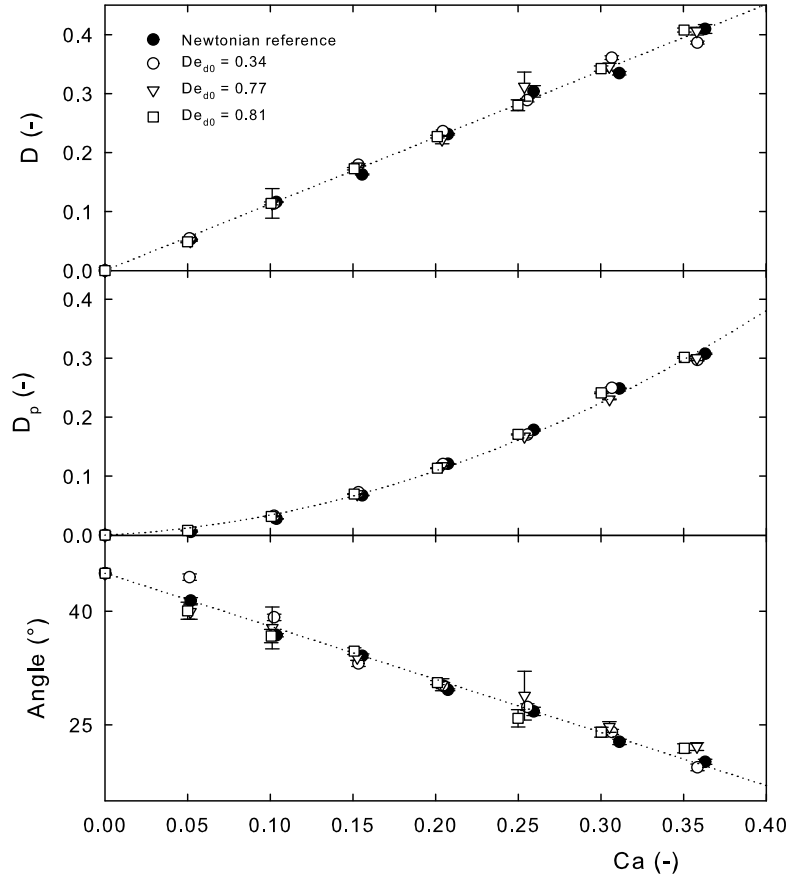


Fig. 8. Steady state droplet deformation and orientation. Open symbols: Shear-thinning viscoelastic branched fluid droplet - Newtonian matrix system (System 2 of table 2) at various \widetilde{De}_{d0} numbers; filled symbols: Newtonian reference system.

served for the Newtonian/Newtonian reference system. Also the orientation of the droplet is similar to that observed for the Newtonian reference system. The drop thus behaves like the Newtonian case, just as the Boger fluid droplet of the previous section, even at capillary numbers beyond the small deformation limit.

Figure 9a shows the steady droplet deformation of the viscoelastic shear-thinning droplet, observed in the velocity-vorticity plane at very high Deborah numbers, *i.e.* smaller droplets. The data are obtained with the Linkam shear cell, studying individual droplets. The steady droplet deformations, measured with the counter rotating setup are also displayed. It is shown that even at very large Deborah numbers (\widetilde{De}_{d0} up to 12), or equivalent imposed shear rates up to 3 s^{-1} , the effect of droplet elasticity and shear-thinning behavior is insignificant, although somewhat more scatter is observed in the Linkam experiments. This becomes more obvious if the results are plotted versus the

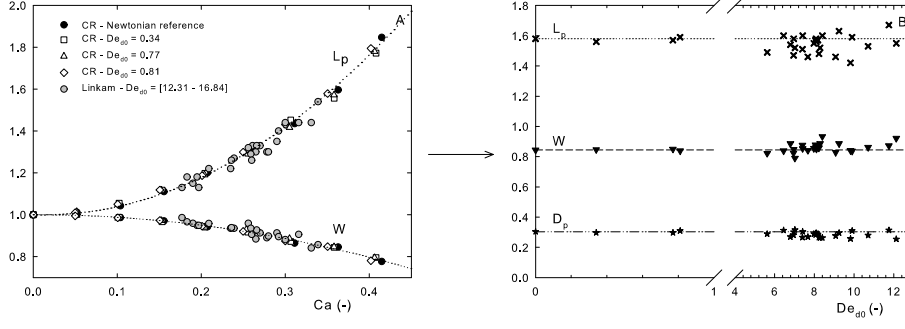


Fig. 9. Steady state droplet deformation of the shear-thinning viscoelastic branched fluid droplet system at various \widehat{De}_{d0} numbers. (a) Top view results as a function of the capillary number; open symbols: counter rotating experiments; filled symbols: Newtonian reference system; gray symbols: Linkam experiments. (b) Top view results at fixed $Ca = 0.35$; lines denote Newtonian steady state deformation.

Deborah number, as shown in figure 9b at a constant capillary number of 0.35. The lines represent the corresponding Newtonian steady deformation. It is obvious that if any effect of droplet elasticity and shear-thinning behavior would be present, it is small and within experimental error.

As in the previous section (e.g., fig. 5), numerical simulations with the Oldroyd-B model at $\beta_{d0} = 0.6$ produce similar droplet history as the Newtonian case. Introducing the shear-thinning by means of a Giesekus model, thereby resembling the rheology fitted with the Ellis model as exactly as possible, does not have a pronounced effect on the resulting steady deformation and orientation. Therefore, the numerical results are omitted here since they mirror those of the previous section. This result is however not surprising, the shear rate inside the droplet is much smaller than the imposed shear rate, and the elastic stresses generated by the recirculation inside the droplet are small, similar to what is shown for the Boger fluid droplet system (see fig. 7).

4.3 Boger fluid matrix system

In figure 10, the steady droplet deformation and orientation for a Newtonian drop - Boger fluid matrix (system 3 of table 2) are plotted as a function of capillary and Deborah number, together with the Newtonian reference system. This clearly shows two primary effects of introducing matrix elasticity, comparable to the results at other viscosity ratios [7, 8]: (i) to promote drop orientation towards the flow direction even at low flow intensities, and (ii) to suppress droplet deformation at higher capillary numbers. These and previous data, do however not allow validation of the non-monotonous dependency of the stationary droplet deformation on matrix viscoelasticity as obtained with the 2D simulations by Yue *et al* [22]. Therefore, the stationary droplet

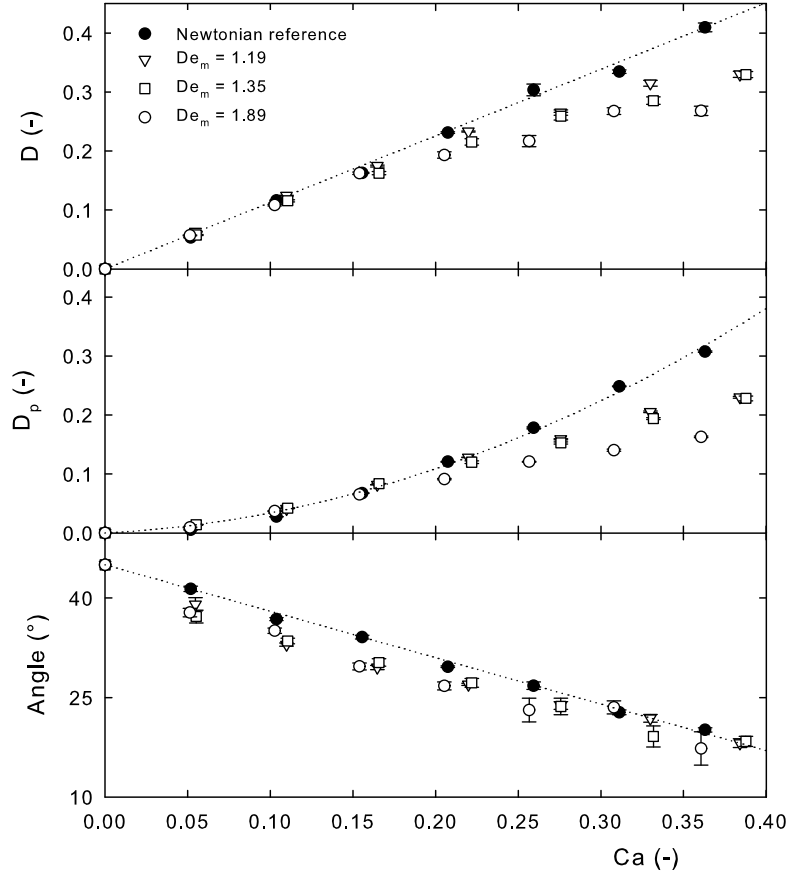


Fig. 10. Steady state droplet deformation and orientation. Open symbols: Newtonian droplet - Boger fluid matrix system (System 3 of table 2) at various \widetilde{De}_m numbers; filled symbols: Newtonian reference system.

deformation and orientation at higher Deborah numbers are addressed in figure 11a .

Figure 11a shows the steady droplet deformation of the Boger fluid matrix system, observed in the velocity-vorticity plane, plotted as a function of the Deborah number (up to 16) at $Ca = 0.35$. It is shown that at $\widetilde{De}_m \approx 2$, the effect of matrix elasticity saturates. The dependency at lower Deborah number (< 2) is similar to that obtained for the BF2 matrix system with a viscosity ratio of 0.75, studied in Verhulst *et al.* [8], and replotted here in figure 11b. Both experiments thus qualitatively yield the same results. At $\widetilde{De}_m < 2$, a decrease in droplet size, or equivalently, an increase in the applied \widetilde{De}_m , results in a decrease of the steady droplet deformation. A sigmoidal dependency on the Deborah number is seen with an inflexion point around $\widetilde{De}_m \approx 1$; the point where non-Newtonian effects are expected to become visible [4].

These experimentally observed trends are at least qualitatively predicted by

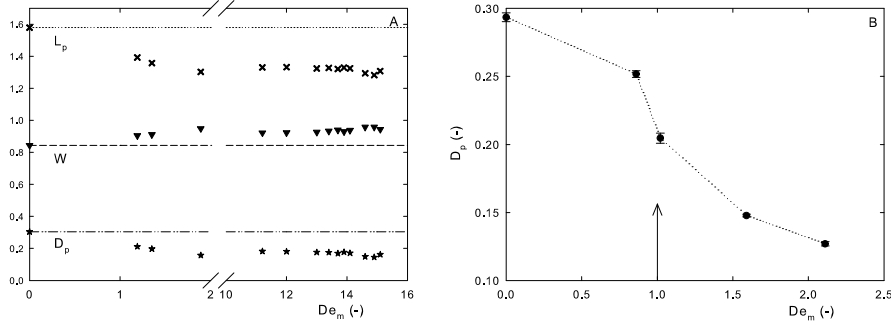


Fig. 11. Steady state droplet deformation of Boger fluid matrix system at various \widetilde{De}_m numbers. (a) Steady droplet deformation observed in the velocity-vorticity plane at $Ca = 0.35$, lines denote Newtonian steady state deformation. (b) BF2 data taken from [8] at $Ca = 0.35$ and viscosity ratio 0.75.

the various phenomenological models presented in literature [8, 9, 10, 11], although they are predicting a monotonous decrease of the droplet deformation with increasing \widetilde{De}_m . Hence, the quantitative prediction of these models at high capillary numbers is less satisfying. Verhulst *et al.* [8] attribute this to the simplicity of the rheological model used by [10]. Hence, in the numerical simulations, the Oldroyd-B model is chosen to describe the rheology of the Boger fluid. Numerical simulations are conducted at $Ca = 0.154$, 0.35, or 0.361, and compared with the data of figures 10 and 11a.

Simulations at low capillary number agree quite well with experimental data, as exemplified by fig. 12 at $Ca = 0.154$ and $\widetilde{De}_m = 1.89$, with $\lambda = 1.5$, $\beta_m = 0.68$. Both deformation and angle of inclination are shown to be closely predicted. Contours of viscoelastic stresses at $\hat{t} = 10, 15, 30$ are also shown in the figure. As time progresses, the area of largest viscoelastic stress moves from the drop tip slightly upwards. In figure 13, simulations with $Ca = 0.154$, and a \widetilde{De}_m increasing from 0 to 4, are shown because the experimental data show saturation in D as the Deborah number increases. For higher Deborah numbers, more mesh refinement is required and the initial transient takes longer. Numerical results in fig. 13 also show little change in the stationary deformation between $\widetilde{De}_m = 0.5$ and 2, and a slight decrease thereafter. Due to spatial periodicity of the drop in the x-direction, contours may enter from the left boundary, as for the case $\widetilde{De}_m = 6$. To obtain fig. 13, higher numerical refinements are used for $\widetilde{De}_m \geq 1.89$.

At higher capillary number, $Ca = 0.35$, simulations with the Oldroyd-B model also show saturation for the higher Deborah numbers, as shown in fig. 14. The upper plot shows the evolution of deformation for $\widetilde{De}_m = 0(\dots) 1$ ($-\cdot-$), 1.89 ($- -$), 4 ($---$), with the Oldroyd-B model. Again, the transients take longer to settle with increasing Deborah number and $\widetilde{De}_m = 4$ is still slightly elongating in the figure. In the lower figure, the deformation values *vs* \widetilde{De}_m show little change for the higher Deborah numbers, as in the 2D simulations of [22]. The

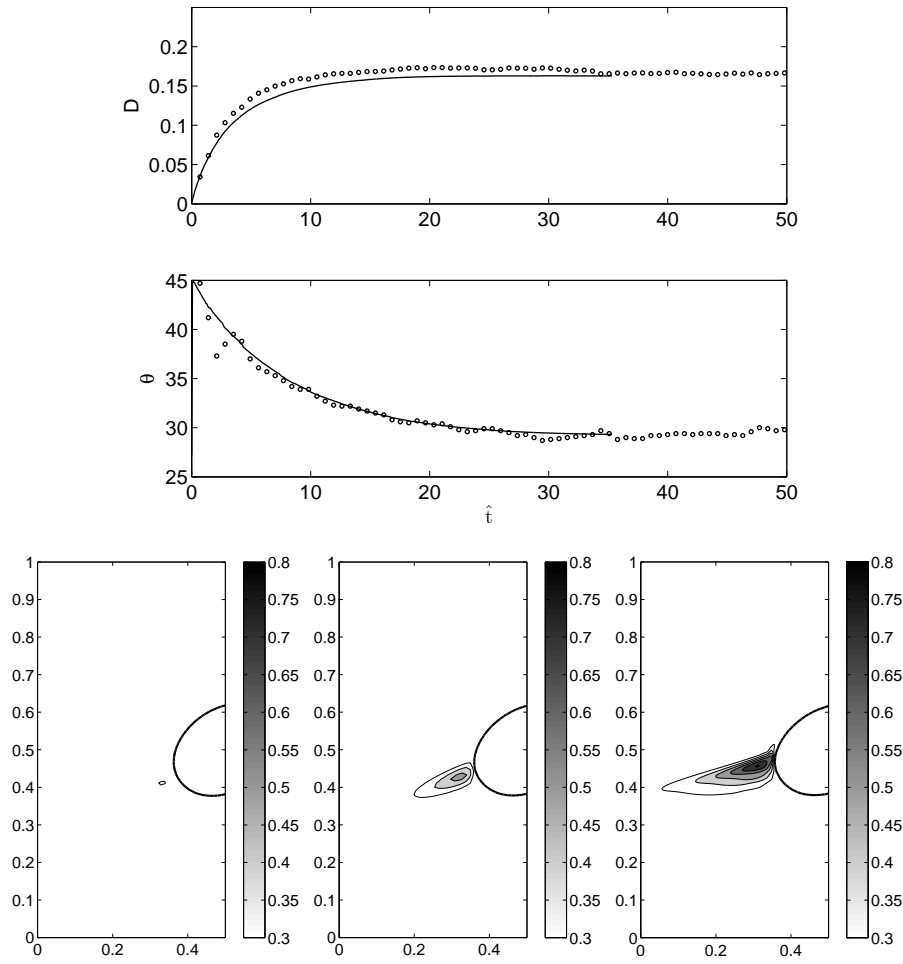


Fig. 12. Newtonian drop in an Oldroyd-B matrix with $Ca = 0.154$, $\widetilde{De}_m = 1.89$, $\lambda = 1.5$, $\beta_m = 0.68$; o experimental data, — VOF-CSF simulation. Drop shapes and viscoelastic stress levels through the vertical cross-section at drop center are shown at $\hat{t} = 10, 15, 30$.

viscoelastic stress contours are shown at $\widetilde{De}_m = 1, 3, 4$ at fixed $\hat{t} = 40$. The location of the maxima lie slightly above the drop tip at the back of the drop rather than at the drop tips, directly at the interface. The streamlines are shown in the $x - z$ cross-section for $\widetilde{De}_m = 4$, and the dividing streamline appears to correspond with the high viscoelastic stresses that come off of the drop in a narrow region.

Figure 15 shows temporal evolution for the particular case of $Ca = 0.35$, $\widetilde{De}_m = 1.89$, for experimental data (o) against the 3D Oldroyd B model (—) and the Giesekus model with $\hat{\kappa}_m = 0.01$. There is a slight decay in deformation in the Oldroyd B simulation for large time, but it essentially saturates to a constant value of deformation and angle. The Giesekus model displays more damping in the deformation and gives a better fit to the data. The data shows additional damping as time progresses, which may reflect the presence of more than one relaxation time. The effect of changing the retardation parameter in

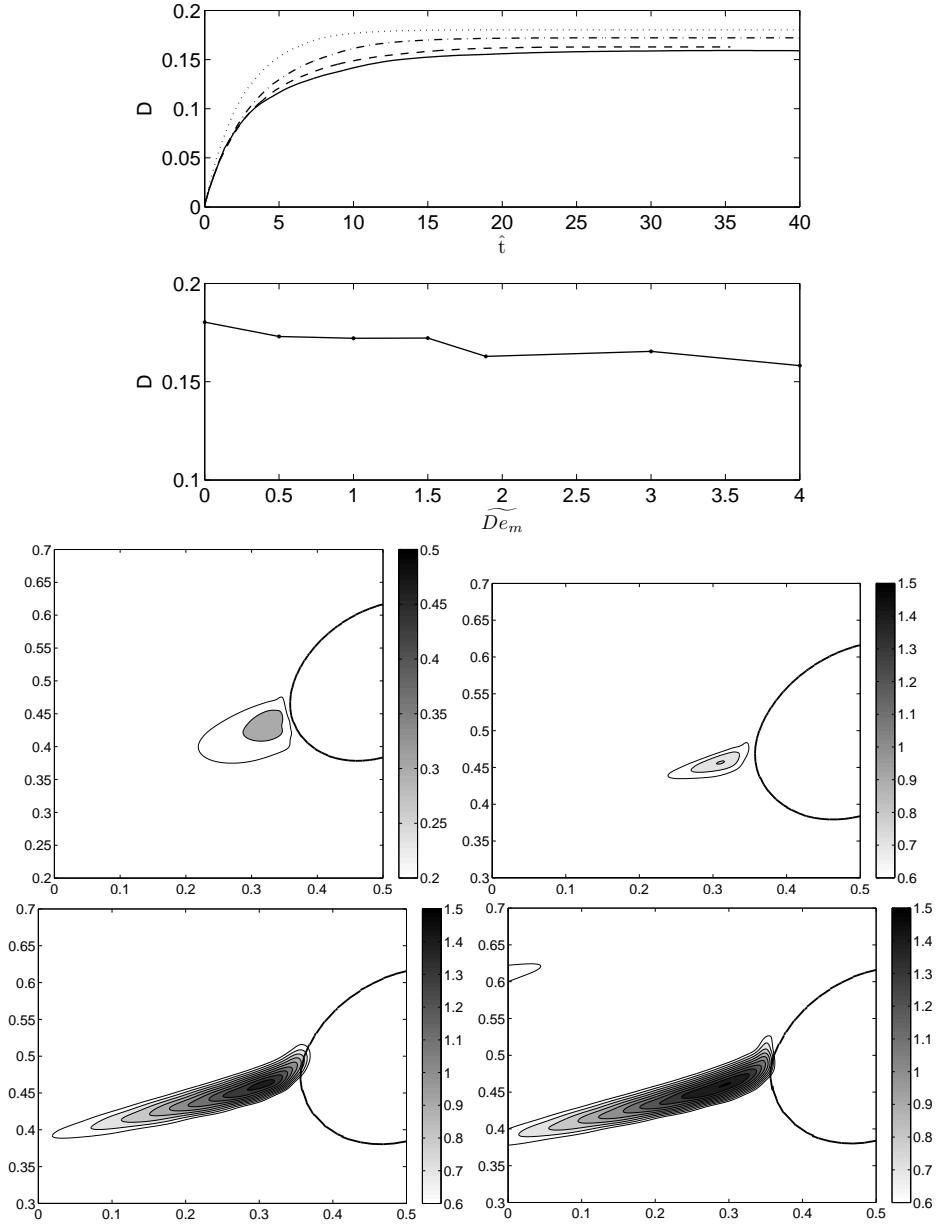


Fig. 13. Newtonian drop in BF2 system, modelled with Oldroyd-B. $Ca = 0.154$. The temporal evolution of deformation for $\tilde{De}_m = 0$ (\dots), 1 ($-\cdot-$), 1.89 ($--$), 4 ($-$), together with the steady state values of deformation D vs \tilde{De}_m . Viscoelastic stress contours for $\tilde{De}_m = 1, 1.89, 4, 6$ at $\hat{t} = 30$.

the 3D simulations is that increasing from $\beta_m = 0.26$ to $\beta_m = 0.8$ decreases the maximum deformation. This provides a check that the value $\beta_m = 0.68$ deduced from table 3 is consistent.

The introduction of shear-thinning in the 3D simulation relieves the overall stress as shown in fig. 16. The left hand column shows the viscoelastic stress growing in intensity, while the addition of $\hat{\kappa}_m = 0.01$ is shown on the right at the corresponding times. The effect of introducing the Giesekus parameter

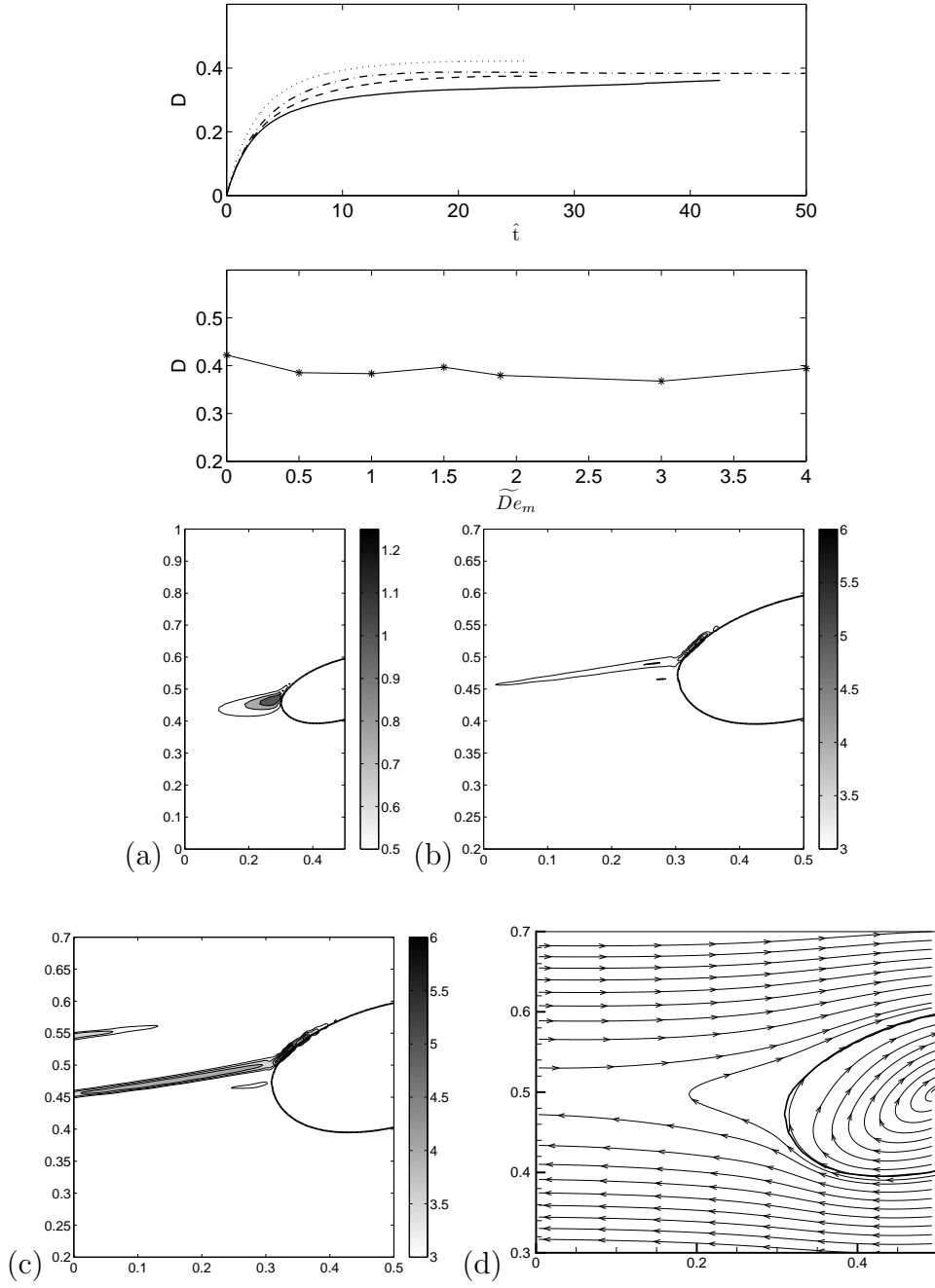


Fig. 14. Newtonian drop in BF2 system, $Ca = 0.35$. The temporal evolution of deformation for $\widetilde{De}_m = 0(\dots)$ 1 ($-\cdot-$), 1.89 ($- -$), 4 ($—$), with the Oldroyd-B model. The lower plot shows the steady state values of deformation D vs \widetilde{De}_m . Viscoelastic stress contour plots in the x-z plane for $\widetilde{De}_m = 1$, (a) at $\hat{t} = 40$, and for $\widetilde{De}_m = 3$ (b), 4 (c), and stream lines for $\widetilde{De}_m = 4$ (d) at $\hat{t} = 30$.

is a slight decrease in the stationary state deformation, and slight increase in the angle of inclination, both of which improve the agreement with the experimental data. However, both settle to constant values rather than the prolonged decay in deformation observed in the data, which may be due to

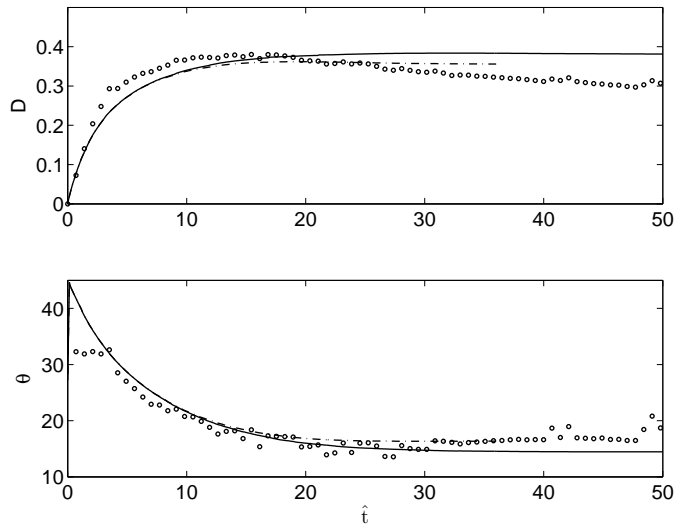


Fig. 15. Newtonian drop in BF2 matrix with $Ca = 0.36$, $\widetilde{De}_m = 1.89$, $\lambda = 1.5$, $\beta_m = 0.68$. Experimental data o; 3D Oldroyd-B model —; Giesekus model with $\hat{\kappa}_m = 0.01$ -·.

multiple relaxation times. The use of, for example, a 5-mode Giesekus model that accurately describes the linear viscoelasticity, the steady shear rheology, and steady and transient extensional rheology could be appropriate. Such a five mode Giesekus description of the BF2 is given in the Appendix. Note that the simulations of [26] at the higher capillary numbers with the Oldroyd-B model overpredicts the viscoelastic stresses, and they use a Giesekus model in order to compare with experimental data. In fact, this is due to an error in their code which has been corrected for this paper.

5 Conclusion

The influence of matrix and droplet viscoelasticity on the steady deformation and orientation of a single droplet subjected to a homogeneous shear flow is investigated microscopically. The viscosity ratio is 1.5 and we focus on capillary numbers around 0.15 and 0.35, outside the range of small deformation asymptotics. Droplet viscoelasticity has hardly any effect on the steady droplet deformation and orientation, even at moderate to high capillary and Deborah numbers. Matrix elasticity, on the other hand, significantly suppresses droplet deformation and promotes droplet orientation, two effects that saturate at high Deborah numbers. This corresponds to decreasing the droplet radius under the same physical conditions. These experimental results are in quantitative agreement with 3D simulations performed with the Oldroyd-B model; accurate results for the higher capillary number are obtained numerically for the first time. The 3D simulations also show for the first time that the stationary value of deformation saturates at higher matrix Deborah numbers,

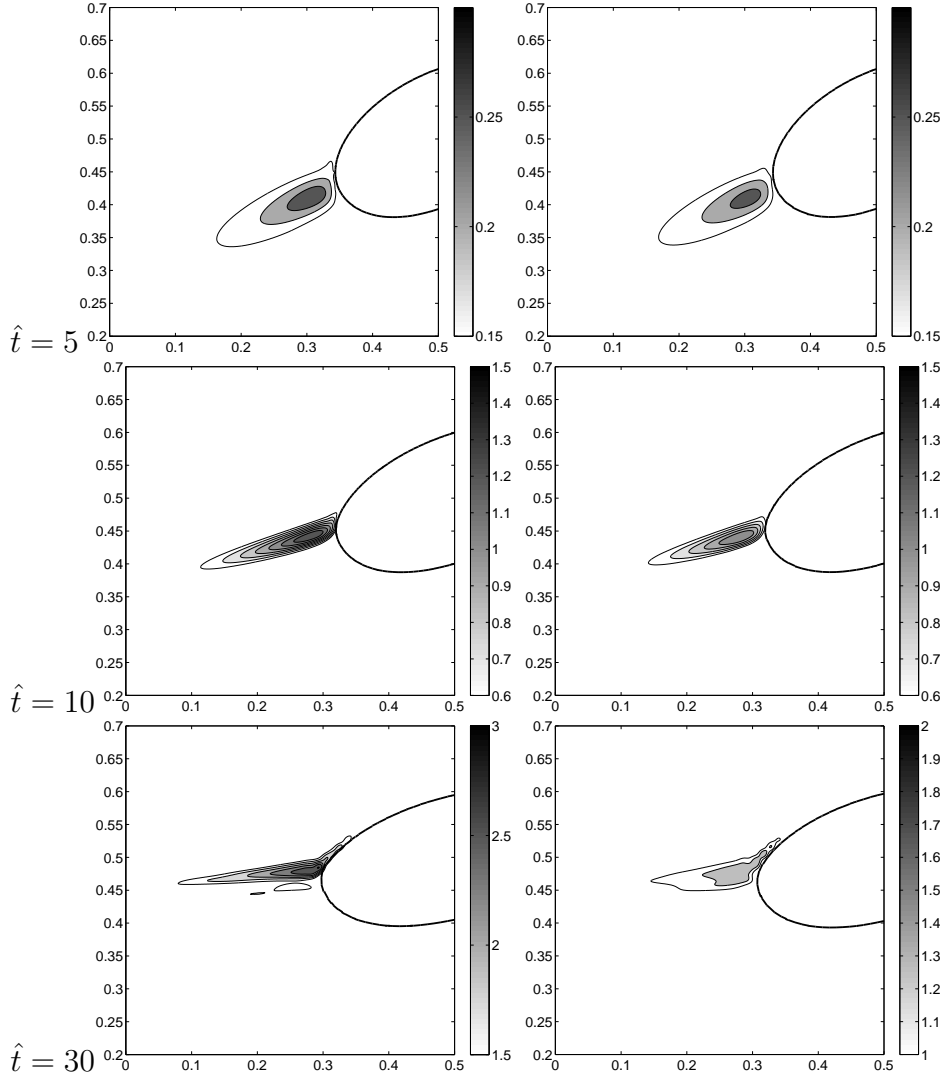


Fig. 16. Viscoelastic stress contours through the vertical cross-section of a drop for fig. 15. Newtonian drop in an Oldroyd-B fluid (left) with $Ca = 0.361$, $\widehat{De}_m = 1.89$, $\lambda = 1.5$, $\beta_m = 0.68$, and Giesekus model with $\hat{\kappa}_m = 0.01$ (right) at $\hat{t} = 5, 10, 30$.

which is also observed in the experimental data. The introduction of some shear-thinning in the matrix fluid by means of a Giesekus model yields the trend that the deformation is lower and the angle is higher, both of which are in the direction of the data. Additionally, the experimental data show a greater decay in stationary state deformation over a longer time scale than is described by the rheological models used in the numerical simulations. This is reconciled by the presence of more than one relaxation time. Indeed, the 5-mode Giesekus model is developed for the BF2 liquid which gives a more accurate prediction of the linear viscoelasticity, the steady shear rheology, and steady and transient extensional rheology.

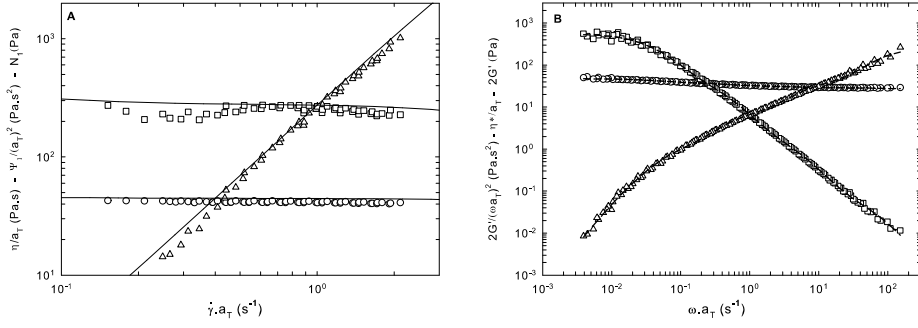


Fig. 1. Rheological characterization of BF2. (a) Steady shear rheology: First normal stress difference Δ , viscosity η , first normal stress coefficient N_1 . (b) Linear viscoelasticity: $2G'/(ωa_T)^2$ \square , dynamic viscosity η' , $2G''$ Δ . Lines are the 5 mode Giesekus model.

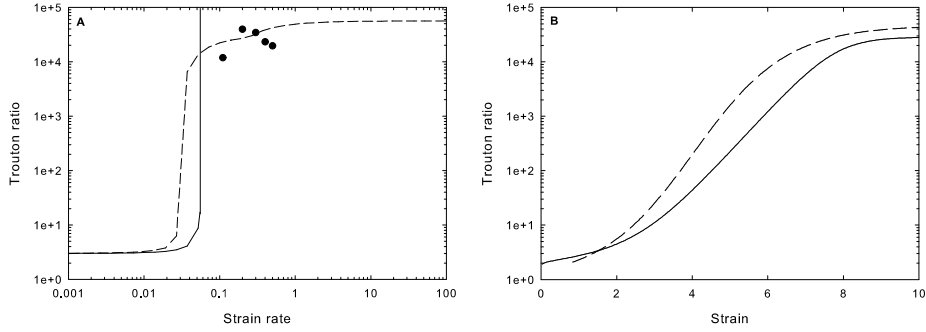


Fig. 2. Rheological characterization of BF2. (a) Steady Trouton ratio. Symbols: experimental data, line: Oldroyd-B model, dashed line: 5 mode giesekus model. (b) Transient Trouton ratio at strain rate of 0.3 s⁻¹. Dashed line: experimental data, line: 5 mode Giesekus model.

APPENDIX

A 5-mode Giesekus model is proposed to describe the rheology of the Boger fluid BF2. It describes the linear viscoelasticity, the steady shear rheology, and steady and transient extensional rheology, as demonstrated in figures .1 and .2. All data are temperature super-positioned with a reference temperature of 25°C. The shift-factors a_T at a temperature of 26°C and 26.4°C are 0.915 and 0.882.

The longest relaxation time is obtained from capillary break-up measurements using a Caber device. The four additional relaxation times and their corresponding partial viscosities are obtained from fitting the linear viscoelasticity with 5 Maxwell modes, as described by Quinzane *et al.* [42]. The $\hat{\kappa}$ -values are obtained from fitting the steady shear data. The resulting 5 relaxation

times and their corresponding partial viscosities and $\hat{\kappa}$ -values; and the solvent viscosity are given in table 5.

This research is supported by NSF-DMS-0456086, NCSA CTS060022, GOA 03/06 and FWO-Vlaanderen for a fellowship for Ruth Cardinaels. The authors would like to thank Prof. Shridhar of Monash University for measuring the extensional rheology of our samples, and Pascal Gillioen & Dr. Jorg Lauger of Anton Paar for their help with the counter rotating device.

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$\Delta x/R_0$	$\Delta t \dot{\gamma}$	D	θ°
1/8	0.0003	0.41	23.0
1/12	0.00014	0.40	23.5
1/16	0.00014	0.39	23.6

Table 1

Tests for accuracy at $\widetilde{De}_d = 2.6$, $Ca = 0.35$, $\hat{t} = 30$, $\lambda = 1.5$, $Re = 0.05$. The fluid pair is system 1 of table 2

	Droplet -Matrix	Droplet phase	Matrix phase	Temp. [°C]	Γ [mN/m]	λ [-]
1	VE-NE	BF2	Saturated Rhodorsil	26.00±0.10	2.2±0.1	1.5
2	VE-NE	BR16	Infineum mix	24.45±0.03	2.65±0.05	1.5
3	NE-VE	Rhodorsil mix 1	BF2	26.40±0.04	2.0±0.1	1.5
4	NE-NE	Rhodorsil mix 2	Parapol 1300	25.50±0.05	2.7±0.1	1.5

Table 2

Blend characteristics at experimental conditions

Polymer	Grade	Temp. [°C]	η_p [Pa.s]	η_s [Pa.s]	Ψ_1 [Pa.s ²]	τ [s]
PIB	Parapol 1300	25.50	83.5
	Infineum mix	24.45	59.1
	BF2	26.00	12.2	25.7	212	8.7
	BF2	26.40	11.7	24.8	197	8.4
PDMS	Rhodorsil mix 1	26.40	53.8
	Rhodorsil mix 2	25.50	125
	Saturated Rhodorsil	26.00	25.2
	BR16	24.45	88.6 ^a	...	317 ^b	1.8

Table 3

Rheology of the blend components at experimental conditions. ^a The tabulated viscosity is the zero shear viscosity obtained from a linear fit with Eq. 8, where $k = 0.4992$ and $n = 0.5430$. ^b The tabulated Ψ_1 is the zero shear first normal stress coefficient obtained from a logarithmic fit with Eq. 8, where $k = 9.3550$ and $n = -0.0988$.

\widetilde{De}_d	L_p	W	D_p	L	B	D	Angle
1.02	1.51	0.88	0.26	1.63	0.71	0.39	23
2.6	1.53	0.85	0.29	1.65	0.7	0.40	23
4	1.58	0.84	0.31	1.7	0.68	0.43	24
12.31	1.64	0.82	0.33	1.75	0.69	0.43	22

Table 4

Numerical simulations for steady state of the elastic drop (system 1, table 2) at $Ca = 0.35$. $\widetilde{De}_d = 1.02$ ($\Delta x = \frac{R_0}{8}$), 2.6 ($\Delta x = \frac{R_0}{16}$), 4 ($\Delta x = \frac{R_0}{8}$), 12.31 ($\Delta x = \frac{R_0}{12}$).

Mode	τ (s)	η_p (Pa.s)	$\hat{\kappa}$
1	49	2.66	0.2
2	16.9	7.43	0.00001
3	2.03	5.82	0.00001
4	0.187	2.69	0.2
5	0.0131	1.39	0.2
Solvent	-	27.2	-

Table 5

Giesekus description with 5 relaxation modes of the BF2 fluid.

- (1) Newtonian reference system with viscosity ratio 1.5 (system 4 of table 2). Experimental data ($- \cdot -$) at $Ca = 0.156$ (lower), 0.363 (upper) are compared with numerical simulations with the boundary integral code of [36] ($--$) and VOF-CSF ($—$).
- (2) Contour plots for viscoelastic stress for a Oldroyd-B drop in Newtonian matrix, $Ca = 0.35$, $\widetilde{De}_d = 2.6$, $\lambda = 1.5$, $\hat{t} = 5$ (row 1), 10 (row 2), 15 (row 3); refinement left to right $\Delta x = R_0/8, R_0/12, R_0/16$.
- (3) Rheological characterization of the viscoelastic fluids at a reference temperature of 25 °C. (a) PIB Boger fluid BF2. First normal stress difference: Δ , viscosity: \diamond , first normal stress coefficient: \circ ; Lines are the Oldroyd-B model. (b) Branched PDMS BR16. Open symbols are dynamic data, $2G'/(\omega a_T)^2$: \circ , dynamic viscosity: \diamond , $2G'$: Δ ; filled symbols are steady shear data, first normal stress coefficient: \bullet , viscosity: \blacklozenge , first normal stress difference: \blacktriangle ; Lines are the Ellis model.
- (4) Steady state droplet deformations and orientation. Open symbols: Boger fluid droplet - Newtonian matrix system (System 1 of table 2) at various \widetilde{De}_d ; filled symbols: Newtonian reference system.
- (5) Side view deformation, 3D VOF-CSF simulation ($—$) and experimental data ($- \cdot -$) at fixed $\widetilde{De}_d = 1.54$, for $Ca =$ (a) 0.14; (b) 0.32, and Newtonian CSF simulation ($--$). The contours for viscoelastic stresses at stationary states are given.
- (6) Steady state droplet deformation of the Boger fluid droplet system at various \widetilde{De}_d numbers. (a) Top view results as a function of the capillary number; open symbols: counter rotating experiments; filled symbols: Newtonian reference system; gray symbols: Linkam experiments. (b) Top view results at fixed $Ca = 0.35$; lines denote Newtonian steady state deformation.
- (7) Steady state viscoelastic stress contours through the vertical cross-section of the drop at $Ca = 0.35$, $\widetilde{De}_d = 1.02$ (a), 2.6 (b), 12.31(c), and stream lines for $\widetilde{De}_d = 12.31$ (d).
- (8) Steady state droplet deformation and orientation. Open symbols: Shear-thinning viscoelastic branched fluid droplet - Newtonian matrix system (System 2 of table 2) at various \widetilde{De}_{d0} numbers; filled symbols: Newtonian reference system.
- (9) Steady state droplet deformation of the shear-thinning viscoelastic branched fluid droplet system at various \widetilde{De}_{d0} numbers. (a) Top view results as a function of the capillary number; open symbols: counter rotating experiments; filled symbols: Newtonian reference system; gray symbols: Linkam experiments. (b) Top view results at fixed $Ca = 0.35$; lines denote Newtonian steady state deformation.
- (10) Steady state droplet deformation and orientation. Open symbols: Newtonian droplet - Boger fluid matrix system (System 3 of table 2) at various \widetilde{De}_m numbers; filled symbols: Newtonian reference system.

- (11) Steady state droplet deformation of Boger fluid matrix system at various \widetilde{De}_m numbers. (a) Steady droplet deformation observed in the velocity-vorticity plane at $Ca = 0.35$, lines denote Newtonian steady state deformation. (b) BF2 data taken from [8] at a $Ca = 0.35$ and a viscosity ratio of 0.75.
 - (12) Newtonian drop in an Oldroyd-B matrix with $Ca = 0.154$, $\widetilde{De}_m = 1.89$, $\lambda = 1.5$, $\beta_m = 0.68$; o experimental data, — VOF-CSF simulation. Drop shapes and viscoelastic stress levels through the vertical cross-section at drop center are shown at $\hat{t} = 10, 15, 30$.
 - (13) Newtonian drop in BF2 system, modelled with Oldroyd-B. $Ca = 0.154$. The temporal evolution of deformation for $\widetilde{De}_m = 0$ (\cdots), 1 ($-\cdot-$), 1.89 ($---$), 4 ($-$), together with the steady state values of deformation D vs \widetilde{De}_m . Viscoelastic stress contours for $\widetilde{De}_m = 1, 1.89, 4, 6$ at $\hat{t} = 30$.
 - (14) Newtonian drop in BF2 system, $Ca = 0.35$. The temporal evolution of deformation for $\widetilde{De}_m = 0$ (\cdots), 1 ($-\cdot-$), 1.89 ($- -$), 4 ($---$), with the Oldroyd-B model. The lower plot shows the steady state values of deformation D vs \widetilde{De}_m . Viscoelastic stress contour plots in the x-z plane for $\widetilde{De}_m = 1$, (a) at $\hat{t} = 40$, and for $\widetilde{De}_m = 3$ (b), 4 (c), and stream lines for $\widetilde{De}_m = 4$ (d) at $\hat{t} = 30$.
 - (15) Newtonian drop in BF2 matrix with $Ca = 0.36$, $\widetilde{De}_m = 1.89$, $\lambda = 1.5$, $\beta_m = 0.68$. Experimental data o; 3D Oldroyd-B model —; Giesekus model with $\hat{\kappa}_m = 0.01$ $-\cdot-$.
 - (16) Viscoelastic stress contours through the vertical cross-section of a drop for fig. 15. Newtonian drop in an Oldroyd-B fluid (left) with $Ca = 0.361$, $\widetilde{De}_m = 1.89$, $\lambda = 1.5$, $\beta_m = 0.68$, and Giesekus model with $\hat{\kappa}_m = 0.01$ (right) at $\hat{t} = 5, 10, 30$.
 - (17) Viscoelastic stress contours through the vertical cross-section for 2D Oldroyd-B simulation in fig. 15. Newtonian drop in an Oldroyd-B fluid (left) with $Ca = 0.361$, $\widetilde{De}_m = 1.89$, $\lambda = 1.5$, $\beta_m = 0.68$, $\hat{t} = 15$, close to stationary state.
- (1) (Appendix) Rheological characterization of BF2. (a) Steady shear rheology: First normal stress difference Δ , viscosity \circ , first normal stress coefficient \square . (b) Linear viscoelasticity: $2G' / (\omega a_T)^2$ \square , dynamic viscosity \circ , $2G''$ Δ . Lines are the 5 mode Giesekus model.
 - (2) (Appendix) Rheological characterization of BF2. (a) Steady Trouton ratio. Symbols: experimental data, line: Oldroyd-B model, dashed line: 5 mode giesekus model. (b) Transient Trouton ratio at strain rate of $0.3 s^{-1}$. Dashed line: experimental data, line: 5 mode Giesekus model.